CANADIAN JOURNAL OF PHILOSOPHY
Volume 27, Number 4, December 1997, pp. 489-512

Kant on Intuition in Geometry[1]

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It's well-known that Kant believed that intuition was central to an account of mathematical knowledge. What that role is and how Kant argues for it are, however, still open to debate. There are, broadly speaking, two tendencies in interpreting Kant's account of intuition in mathematics, each emphasizing different aspects of Kant's general doctrine of intuition. On one view, most recently put forward by Michael Friedman,[2] this central role for intuition is a direct result of the limitations of the syllogistic logic available to Kant. On this view, Kant's reasons for introducing intuition are taken to be logical or mathematical, rather than

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1 All references to Kant's writings, except references to the *Critique of Pure Reason*, are given by volume and page number of the Akademie edition of *Kant's gesammelte Schriften*, Georg Reimer, 1900-; the *Critique of Pure Reason* is cited by the standard A and B pagination of the first (1781) and second (1787) editions respectively. Except where noted, the translations are from the following editions:
Immanuel Kant's Critique of Pure Reason, Norman Kemp Smith, trans. (London: MacMillan 1986).
Kant: Theoretical Philosophy 1755-1790, David Walford in collaboration with Ralf Meerbote, ed. and trans. (Cambridge: Cambridge University Press 1992).
Kant: Lectures on Logic, J. Michael Young, ed. and trans. (Cambridge: Cambridge University Press 1992).
'On a Discovery According to Which Any New Critique of Pure Reason has been made Superfluous by an Earlier One,' Henry Allison, trans., in *The Kant-Eberhard Controversy* (Baltimore and London: Johns Hopkins University Press 1973).
Prolegomena to any Future Metaphysics, James W. Ellington, trans., in *Immanuel Kant: Philosophy of Material Nature* (Indianapolis: Hackett 1977).

2 *Kant and the Exact Sciences* (Cambridge, MA: Harvard University Press 1992), chs. 1 and 2.

philosophical. The other tendency, which I shall try to develop here, emphasizes an epistemological or phenomenological role for intuition in mathematics arising out of what may loosely be called Kant's 'anti-formalism.'

This paper, which focuses specifically on the case of geometry, falls into two parts. First, I consider Kant's discussion of intuition in the Metaphysical Exposition of the concept of space. The goal is to show that there are elements of this discussion for which Friedman's reading cannot account, and to set out an alternative reading which does account for them. In the second part, I attempt to make clear what the philosophical — as opposed to logical — role for Kant's notion of intuition is in his account of our knowledge of geometry.

My goals in this paper are quite modest: I don't intend to defend Kant's philosophy of geometry, nor do I attempt to give a complete account of *how* Kant's notion of construction in pure intuition is supposed to fill the role that Kant saw for it. I do, however, wish to suggest some ways in which Kant was dealing with issues which are of importance to philosophers of mathematics independently of the limits of the logic and geometry of his time.

I Infinity and Polyadic Quantification

One of Kant's reasons for taking geometry as a paradigmatic body of synthetic a priori knowledge is that geometrical *reasoning* cannot proceed 'analytically from concepts.' Rather, the geometer proceeds by constructing concepts in pure intuition. As Friedman puts it, for Kant, the postulates and definitions of Euclidian geometry do not entail the theorems by logic alone. For example, the Euclidian proof that an equilateral triangle can be constructed from any given line segment requires the existence of a point of intersection of two constructed circles. But Euclid's system does not guarantee the existence of such a point; there are models of the axioms in which such a point does not exist. One thing that is lacking is a continuity axiom, the formulation of which requires polyadic quantification. Although Euclid's second postulate states that straight line segments are produced 'continuously,' this is not sufficient to carry out the proof by merely logical means since this postulate could not, in Kant's time, be logically analyzed.

According to Friedman, Kant's appeal to intuition is primarily the result of this limitation of his monadic logic. For Kant only the indefinite iterability of constructive processes, such as bisecting a line, can guarantee the existence of the infinitely many points required for the denseness of line segments; only appeal to the motion of a point guarantees the existence of the limit points required for true continuity. The general problem is the inability of Kant's monadic logic to deal with infinity. Only by means of quantifier dependence over relations can the existence of an infinity of objects be deduced explicitly by logic alone. As Friedman explains, the modern approach to geometry proceeds by means of an axiomatized theory of dense linear order without endpoints, where the denseness condition is expressed by the axiom $\forall a \forall b \exists c\ (a < b \rightarrow (a < c < b))$. Because of the limited expressive power of the monadic logic at his disposal, however, Kant cannot 'represent or capture the idea of infinity formally or conceptually' but only 'intuitively — by an iterative process of spatial construction' (Friedman, 63). Pure intuition gives a method — the iteration of intuitive constructions — for doing the work of the existential quantifiers in formulas expressing the property of denseness. Whereas in modern axiomatic treatments, such a formula 'guarantees' the existence of a point between any point a and any point b, for Kant, the existence of such a point is guaranteed by a fact about our intuitive capacities, i.e., the fact that we can bisect any given line segment, thereby actually *constructing* the required point. Friedman seems to suggest that this is why Kant views the truths of geometry as synthetic rather than analytic: with reference to the procedure of generating new points by the iterative application of constructive functions, he says that

> *Since* the methods involved go far beyond the essentially monadic logic available to Kant, he views the inferences in question as synthetic rather than analytic. (Friedman, 65, my emphasis)

But is it merely the syntheticity of the *inferences* of geometry that accounts for the syntheticity of the *truths* of geometry? Does Kant's claim that the truths of geometry are grounded in pure intuition therefore result from the limitations of monadic logic? I shall argue that it is not merely or even primarily the syntheticity of inferences in geometry that interests Kant.

II Infinity and Intuition in the Metaphysical Exposition

While Friedman's analysis of this inadequacy of monadic logic is by now both familiar and uncontroversial, the suggestion that Kant brings in pure intuition *solely* to fill this gap is not. In the Metaphysical Exposition of the concept of space of the *Critique of Pure Reason* (A22/B37ff), Kant gives an argument — purportedly independent of geometrical concerns — for the claim that the representation of space as an infinite given magnitude must be intuitive since a general concept of space is inadequate for such a representation. (This of course assumes Kant's exhaustive division of representations into concepts and intuitions.) Kant's explicit claim here is not merely that features of our representation of space cannot be captured formally for the purposes of rigorous geometrical proof, but that space cannot be first given conceptually: rather all concepts of space are derived from the original intuitive representation. This suggests that the argument for the intuitive nature of our representation of space is independent of any geometrical concerns, and thus independent of the inferential role for intuition in geometry. The syntheticity of geometry, as the science of space, would then follow straightforwardly from the intuitive nature of our representation of space regardless of the inferential role for intuition. But Friedman attempts to circumvent this reading by showing that Kant's argument for the inadequacy of the general concept for the representation of space *also* depends at bottom on the limitations of monadic logic.

To establish this connection, Friedman focuses on the passage at B40 of the Transcendental Aesthetic, which concludes the argument that the original representation of space is an a priori intuition, and not a concept. Here Kant describes two features of concepts which are supposed to explain why the representation of space cannot be a concept. First, every concept is a representation which is 'contained in an infinite number of different possible representations' and which therefore contains these representations *under* itself. But second, no concept 'contains an infinite number of representations *within* itself.' Thus, since space is a representation containing an infinite number of representations within itself (i.e., 'all the parts of space coexist *ad infinitum*'), it must be an intuition.

The key question, according to Friedman, is why Kant thinks that a concept cannot 'contain an infinite number of representations within itself.' Before we can answer this, we have to consider what Kant means by a concept containing representations *within* itself, as opposed to *under* itself. He explains this in his lectures on logic. A concept contains *under* itself those concepts which are obtained from it by adding differentia. For example, the concepts 'plant,' 'oak tree,' and 'chestnut' are contained under the concept 'body.' On the other hand, a concept contains within itself those concepts which are component parts of its definition: for example, the concept 'oak tree' contains the concepts 'plant' and 'body.' These are the concepts contained *within* the original concept. This should help to clarify the claim we are concerned with here. A concept may contain under itself infinitely many concepts in that an indefinite number of differentia may be added to it. According to Kant:

> The *conceptus infimus* cannot be determined. For as soon as I have a concept that I apply to *individua*, it would still be possible for there to be still smaller differences among the *individua*, although I make no further distinction. (Ak.24:911)

Any given concept, on the other hand, can contain *within* itself only finitely many concepts because, according to Friedman, a finite mind could never grasp infinitely many concepts.[3] Intuitions, however, are different. They are divided by the introduction of limitations, which can proceed without limit. What Kant calls the 'limitlessness of the progression of intuition' is supposed to allow an intuitive representation to contain within itself an infinite number of representations (A40). It thus follows that since our representation of space contains within itself an infinite number of representations, it must be an intuition.

This argument for the claim that the original representation of space is not a concept then depends on facts about the logical structure of concepts and intuitions. We can begin to see how Friedman might argue that these facts are artifacts of Kant's limited logical resources. The important fact here is the inability of concepts to contain infinitely many representations within themselves. How is this connected to the limitations of monadic logic? Friedman takes the problem here to be that concepts 'cannot be conceived as the conjunction of an infinite number of constituent concepts' (Friedman, 67). These constituent concepts are given by *monadic* predicates. But given k monadic predicates, we can only distinguish $2k$ distinct types of objects (objects that are P_1 & P_2 &...& P_k, objects that are not-P_1 & P_2 &...& P_k, objects that are P_1 & not-P_2 &...& P_k, etc.) It is for this reason that no concept can represent an infinity of objects. Polyadic quantification, however, allows us to overcome Kant's restriction on concepts: by allowing us to bypass infinite conjunction, it allows finite intellects to grasp infinitely many representations in one concept and thus to 'describe' infinitely many objects by presenting formulas with only infinite models. Just as polyadic logic enables us, by means of the ancestral relation, to give an abstract characterization of the series of natural numbers, it seems to pave the way for a conceptual representation of space as an infinite magnitude by means of a theory of dense linear order, for example. This, of course, brings up the question of the nature of the infinity of space, to which we shall return below.

But now let's consider the role that this general fact about the way that concepts involve infinity plays in Kant's *overall* argument about space.

3 As an anonymous referee pointed out, this is not Kant's argument for the claim that a concept can contain within itself only finitely many concepts. He argues rather that it is because there is a highest genus (Ak.9:59), that is, the concept of an object (Ak.24:755) or *something* (Ak.24:911), which we can arrive at it in a finite number of steps by omitting everything (ibid.). But it doesn't seem to follow from the existence of a highest genus that we can arrive at it in a finite number of steps without some additional assumption about the finiteness of our minds.

What Friedman has done so far is to recast the argument at B40, which emphasizes the *infinite* nature of our representation of space, in order to argue that the inference from that feature of space to the intuitive nature of our representation of it depends on the limitations of monadic logic. But of course, this is not Kant's only argument here. Prior to the passage at B40, he also argues that space must be a pure intuition (rather than a discursive concept) because (a) it is a singular individual, and (b) the parts cannot precede the singular space as its constituents. This argument appears to be independent of considerations about infinity, but Friedman nonetheless recasts it in such a way that it too depends on the limitations of monadic logic. I want to turn to this argument now.

Kant's presentation of this first argument at B39 begins with the assertion that different spaces are only parts of one unique space, and these parts cannot precede the whole, but 'can be thought only as in it.' Thus space cannot be a general concept because, first, the relation between particular spaces and the individual space is one of parts to whole, and not of instances to concept; and secondly, one cannot arrive at the singular individual space by assembling all the instances of the general concept. On the contrary, particular spaces are arrived at only by means of 'limitation' of the individual space, and thus the general concept of which particular spaces are instances is obtained only *after* the introduction of limitations. As Kant puts it, 'space is essentially one; the manifold in it, and therefore the general concept of spaces, depends solely on limitations.' Thus an intuition of the singular individual space must underlie the general concept of space.

Kant's argument here turns on the claim that the parts of space are obtained only by an intuitive procedure of limitation. Exactly why can't we take the parts of space to be given as instances of the concept 'x is a space'? At this point, Friedman thinks that considerations about the infinity of our representation of space resurface. In answer to the question why intuition must play an essential role in our knowledge of geometry, he says:

> what is required for establishing the intuitive character of our representation of space is not simply the fact that space consists of parts, but rather — as geometry demonstrates — the fact that it consists of an *infinite number* of parts. (Friedman, 70)

According to Friedman, the reason Kant must appeal to an intuitive procedure of introducing limitations is that, as the argument at B40 goes on to show, this is the only way he can account for infinite divisibility, since 'no mere monadic concept can possibly capture this essential feature of our representation of space' (ibid.), whereas the unbounded iterability of the intuitive act of limitation presumably does capture this feature. Moreover, it is for this reason that Kant asserts the singularity of space: only a singular individual is divided by the introduction of limitations. Thus the intuition of the singular individual space must be prior to the general concept.

But as we've seen from Friedman's analysis of B40, on this view the fact that no concept can capture the infinite nature of space is simply an artifact of Kant's logic. The relation between the parts of space and space could be represented, according to Friedman, as we can now represent the relation between points and a line by a polyadic theory of dense linear order. Taking points as primitive, we view the axioms as a 'conceptual representation' of the line with the variables ranging over points: an individual line can then be viewed as composed of infinitely many points. Similarly, we could view space as composed of the infinitely many parts of space taken as instances of the concept '*x* is a space,' and thus, in Kant's words, 'preceding the single space as its constituents.' The intuitive procedure of introducing limitations need not be appealed to in order to exhibit the infinite divisibility of space.

Crucial to Friedman's argument here is the assertion that 'Kant's claim of priority for the singular intuition *space* rests on our knowledge of geometry' (ibid.). As we have just seen, Friedman in effect collapses the third and fourth arguments of the B edition of the Aesthetic. He wants to say that the reason that Kant insists on the claim in the third argument that the parts of space are introduced only by limitation, is that, because of the structural features of concepts described in the fourth argument, only an intuitive representation can contain in it an infinite number of representations, as our representation of space must do in order to account for the infinite divisibility as revealed to us by geometry. Indeed, it is the unbounded iterability of specified constructive procedures underlying the proof-procedure of Euclidian geometry which 'makes the idea of infinity, and therefore all "general concepts of space," possible' (Friedman, 71). The infinity of space is, for Friedman's Kant, simply a formal feature of geometry.

Three considerations cast suspicion on this reading of the Metaphysical Exposition.

First of all, Kant presents the third argument as independent of considerations of infinity, and indeed as independent of the fourth argument.

Second, by basing both arguments on the infinite divisibility revealed by geometry, Friedman goes against Kant's explicit assertion in the *Prolegomena* §4 that in the *Critique*, he is pursuing the 'synthetic method' which is 'based on no data except reason itself, and which therefore seeks, without resting upon any fact, to unfold knowledge from its original germs.' This is opposed to the analytic method followed in the *Prolegomena*, which rests upon something 'already known as trustworthy,' that is, mathematics and natural science, from which we 'ascend to the only conditions under which it is possible.'

Third, and more specifically, Friedman's reconstruction doesn't seem to respect the similar distinction within the *Critique* between the Transcendental and the Metaphysical Expositions, where the former proceeds by considering what must be the case in order for our knowledge of geometry to be possible, whereas the latter considers 'that which exhibits the concept as *given a priori*' (A23/B38), as a condition of the possibility of experience.[4]

These considerations are, of course, not decisive, not least because it's not completely clear how to understand these latter two distinctions. I will try in what follows to present an alternative reading of the Metaphysical Exposition, which in turn may shed some light on how Kant understands these distinctions. What is at issue here, I think, is what Kant takes as data for the argument of the Metaphysical Exposition. Friedman's reconstruction of the argument begins from the premise — taken from geometry — that space is infinitely divisible and proceeds by means of the necessary appeal to the procedure of limitation to the conclusion that space is singular. I want to suggest that the order of explanation is roughly the reverse: because space is singular, it is divisible only by limitation, and it follows from this that it is infinitely divisible. I will argue that Kant does not claim that the unlimited progression of intuition allows us to *capture* the antecedently-given infinity of geometrical space, but rather that it is that in virtue of which geometrical space is infinite.[5] So whereas Friedman takes the essential feature of space underlying the arguments of the Metaphysical Exposition to be infinite divisibility as revealed by geometry, I want to suggest rather that the essential feature is the uniqueness and boundlessness of space as a condition for the possibility of experience, from which the infinity of geometrical space is supposed to follow. This is a claim about the space of *experience*, how space is given to us independently of and indeed prior to geometry, and it is this experience which serves as data for Kant's synthetic method.

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4 Friedman's disregard of these two distinctions reflects his attempt, which I mentioned earlier, to show that Kant's reasons for introducing intuition were primarily mathematical, and not philosophical.

5 For this reason, it strikes me as somewhat misleading to talk, as Friedman does, of 'capturing' the infinite divisibility of space, as this suggests that the feature is available to us independently of the means of capturing it, i.e., the representation of space. It's not clear to me that Kant would accept this, and indeed, this may be at the heart of the matter, as we shall see below.

What is at issue here is whether the Metaphysical Exposition constitutes some kind of *justification* or *foundation* for geometry. I want to suggest that Kant distinguishes the metaphysical treatment of space from the geometrical treatment of space, and that the former is prior to the latter, and in fact grounds it. This comes out clearly, I think, in the following passage from Kant's discussion from 1790 of Kästner's treatises, where he distinguishes these two treatments of space (Ak.20:419-20). The task of metaphysics, he says, is to show 'how one *has* the representation of space'; here, space is considered 'as it is *given*, before all determinations.' Geometry, however, 'teaches how one *describes* a space, i.e., can present it a priori in a representation'; here, space is considered 'as it is generated [*gemacht*].' Moreover, in metaphysics, space is '*original* and only one (single) *space*'; in geometry, it is *derived* and there are (many) *spaces*.' The point then, is that the generation of geometrical spaces presupposes a metaphysical space in which they are generated. Kant continues:

> The geometrician, however, in agreement with the metaphysician, and as a consequence of the fundamental representation of space, must confess that these spaces can only be thought as parts of the one original space.

Whereas on Friedman's reading, the infinity of space even in the Metaphysical Exposition rests on the indefinite iterability of Euclidian operations — in the same way that the infinity of the natural numbers rests on 'our capacity successively to iterate any given operation' (Friedman, 126) — Kant is quite clear here that this is not the case. The infinite given space described in the Metaphysical Exposition is not constructed by the iteration of Euclidian operations; on the contrary, the iteration of those operations presupposes the infinite given space of the Exposition:

> For the representation of space (together with that of time) has a *peculiarity* found in no other concept; viz., that all spaces are only possible and thinkable as parts of one single space, so that the representation of parts already presupposes that of the whole. Now, if the geometer says that a straight line, no matter how far it has been extended, can still be extended further, this does not mean the same as what is said in arithmetic concerning numbers, viz., that they can be continuously and endlessly increased through the addition of other units or numbers. In that case the numbers to be added and the magnitudes generated through this addition are possible *for themselves*, without having to belong, together with the previous ones, as *parts of a magnitude*. To say, however, that a straight line can be continued infinitely means that *the space in which I describe the line is greater than any line which I might describe in it*. Thus the geometrician expressly grounds the possibility of his task of infinitely increasing a space (of which there are many) on the original representation of a single, infinite, subjectively given space. This agrees very well with the fact that the geometrical and objectively given space is always finite. For it is only given in so far as it is generated. To say, however, that the metaphysical, i.e., original, but merely subjectively given space, which (because there is not a plurality of them) cannot be

brought under any concept capable of construction, but which still contains the ground of the possibility of all geometrical concepts, is infinite, means only that it consists in the pure form of the mode of sensible representation of the subject, as an a priori intuition, and therefore as a singular representation, in which the possibility of all space, proceeding to infinity, is given.[6]

There are two important points to emphasize from this passage at this point. First, the possibility of geometrical construction is grounded in metaphysical space; the geometrical properties of space are therefore dependent on, derived from, the original representation of space, not, as Friedman would have it, the other way around. As Kant says, the geometer 'grounds the possibility of his task' on the original representation of space. It is therefore not the case that Kant can only represent the idea of infinity 'by an iterative process of spatial construction' (Friedman, 63). However far we extend the line by this process, there is more space into which the line could be extended further; the infinite extendibility of the line by this process *depends* on this latter fact. This suggests, second, that Kant has independent grounds for asserting the infinity and singularity of this original representation, in particular since 'geometrical space is always finite.' In other words, geometry does not — indeed, *cannot* — provide the data for Kant's argument in the Metaphysical Exposition: rather, the metaphysician considers space 'as it is *given*, before all determinations.'

What then *are* Kant's grounds for asserting the infinity and singularity of space? What does he mean by space 'as it is given'? Kant provides some indication of an answer in the passage we have just been looking at. With respect to the singularity of space, it is precisely the peculiarity of the representation of space (and that of time) that 'all spaces are only possible and thinkable as parts of one single space.' For this reason, 'the representation of parts already presupposes that of the whole.' With respect to the infinity of space, he says that 'one can only view as *infinite* a magnitude in comparison to which any specified similar [*gleichartige*] magnitude is equal only to a part' (Ak.20:419). This recalls Kant's argument for the intuitive nature of our representation of space in the Transcendental Aesthetic, considered above, and in §15B of the Inaugural Dissertation:

The concept of space is a singular representation comprehending all things *within itself*, not an abstract common notion containing them *under itself*. For what you speak of

6 Ak.20:419-21. Friedman himself cites this passage in this connection in an unpublished paper entitled 'Geometry, construction and intuition in Kant and his successors.'

> as several places are only parts of the same boundless space, related to one another by a fixed position, nor can you conceive to yourself a cubic foot unless it be bounded in all directions by the space that surrounds it.

Parsons has suggested that Kant, both here and in the Aesthetic, is making claims 'of a phenomenological character' about 'space as experienced': in this case, that 'places, and thereby objects in space, are given in a one space, therefore with a "horizon" of surrounding space.'[7] On this reading, the uniqueness of space is indicated by the fact that particular spaces are given in one all-encompassing space. The boundlessness of space is shown by the fact that any given space, however large, is given as bounded by more of the same. Similarly, particular spaces are given only as limitations of the all-encompassing space. These latter two facts seem to me to underlie Kant's claim that the progression of intuition is limitless, and indeed to give sense to the idea that such a succession of intuitions can proceed without limit in both directions. That this is the sense in which space, for Kant, is *given* as infinite seems clear from the parallel passage in the argument about time, where he says that

> the infinitude of time signifies nothing more than that every determinate magnitude of time is possible only through limitations of one single time that underlies it. The original representation, *time*, must therefore be given as unlimited. (A32/B48)

That the infinite divisibility of space is supposed to follow from this is indicated in a passage from the Anticipations of Perception:

> The property of magnitudes by which no part of them is the smallest possible, that is, by which no part is simple, is called their continuity. Space and time are *quanta continua* because no part of them can be given save as enclosed between limits (points or instants), and therefore only in such fashion that this part is itself again a space or a time. Space therefore consists solely of spaces, time solely of times. Points and instants are only limits, that is, mere positions which limit space and time. But positions always presuppose the intuitions which they limit or are intended to limit; and out of mere positions, viewed as constituents capable of being given prior to space or time, neither space nor time can be constructed. (A169/B211)[8]

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7 Charles Parsons, 'The Transcendental Aesthetic,' in Paul Guyer, ed., *The Cambridge Companion to Kant* (Cambridge: Cambridge University Press 1992), 72

8 Kant describes the continuity of time in the same terms in §14.4 of the Inaugural Dissertation: 'any part whatever of time is itself a time. And the things which are in time, simple things, namely *moments* are not parts of time, but *limits* with time between them.' Curiously, he does not provide a parallel argument there for the continuity of space, but he does claim in a footnote that this is easily demonstrated (Ak.2:403).

This, I think, explains the sense in which, for Kant, the geometer grounds the possibility of his task on the original representation of a single, infinite, subjectively given space.

I have tried to argue against Friedman's claim that ultimately Kant's argument for the intuitive nature of space rests on our knowledge of geometry and that 'our cognitive grasp of the notion of space is manifested, above all, in our geometric knowledge' (Friedman, 70). I have argued that, on the contrary, in the Metaphysical Exposition Kant is providing *independent* grounds for taking the space in which objects are given to be as geometry describes it.[9] The Transcendental Exposition then shows that this analysis of our representation of space *also* accounts for our knowledge of geometry, and indeed is the only analysis which could account for it.[10]

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9 This argument also constitutes a reply to an objection first raised by Parsons ('Infinity and Kant's Conception of the "Possibility of Experience,"' *Philosophical Review* 73 [1964]. Reprinted in *Mathematics in Philosophy* [Ithaca: Cornell University Press 1983] 95-109) that Kant's attempt to explain how we have synthetic a priori knowledge of certain features of space — in particular, the infinite divisibility of space — is ad hoc. The problem is that Kant limits the extent of our synthetic knowledge to objects of possible experience; but, Parsons argued, when we try to give a concrete intuitive meaning to the notion of possible experience, we see that the limits of possible experience are narrower than the extent of the geometric knowledge which Kant wants to account for. The only alternative is to define the possibility of experience by what, on *mathematical* grounds, we take to be the form of our intuition. But if the form of intuition is determined by our knowledge of geometry, it cannot be called upon to provide an *explanation* of that knowledge. On the present account, however, the nature of the form of intuition is determined independently of geometry, and thus *can* be called upon to explain our knowledge of geometry, as I have just argued. This is not to say that Kant's argument for the infinity of space is successful. All I have tried to show is that Kant sought to explain our knowledge of geometry in this way, not that he succeeded in doing so. That would require a much more detailed analysis of the argument of the Exposition than has been given here.

10 It also seems to me that taking the 'cognitive grasp' determining the nature of our representation of space as given primarily by geometry renders uninteresting any argument against the Wolff-Leibniz view of geometry that, as Friedman himself makes clear, Kant opposed throughout his writing, that is, the view that geometrical concepts are, in some sense, 'imaginary.' Against this, Kant is determined to show how geometrical concepts are grounded in the world of experience. But then, to assume that the essential features of our representation of space are to be determined by what is required for geometry begs the question. In particular, Friedman's focus on infinite divisibility seems misplaced, since it was the infinite divisibility of geometrical space which resulted in what Leibniz called 'the labyrinth of the continuum,' which in turn contributed to the Leibnizean view that geometric space is ideal (see, e.g., *Die philosophischen Schriften von Gottfried Wilhelm Leibniz*. Hrsg. C.

I hope that I have shown how, for Kant, the geometrical treatment of space presupposes the representation of space as presented in the Metaphysical Exposition. But does it follow from these considerations that there is a larger role for intuition *in* geometry than that which Friedman gives to it? This is the question I want to turn to now. While I've tried to show above that Kant has grounds for holding that the 'original representation' of space must be a pure intuition other than for the representation of the infinite features of geometrical space, we might take Friedman's argument to have established the weaker claim that Kant would agree that *geometry* could proceed purely conceptually if it weren't for the limitations of monadic logic. But I think, partly for reasons stemming from the phenomenological considerations adduced in the Metaphysical Exposition together with Kant's claims about the nature of geometric *evidence*, that this is not the case.

III Intuition and Geometry

The basic claim I want to defend here is that, as Parsons has put it, even if Kant had recognized the possibility of a purely conceptual description of mathematically infinite magnitude, 'there would be the further question of constructing it' (Parsons, 'The Transcendental Aesthetic,' 71). So the question now is this: what is added in the geometrical construction in pure intuition that is not and could not be contained in the concept?

For Friedman, as we've seen, the role of construction in pure intuition is inferential: Kant's limited logical resources are not sufficient for carrying out certain kinds of inferences, in particular, those involving infinity, and so intuition serves as an extra-logical form of inference. But given the expressive power of polyadic quantificational logic, these extra-logical forms of inference are no longer required, and thus neither is pure intuition.

There is, of course, an alternative interpretation of Kant's claims about the role of intuition in mathematics, one developed in various ways (and with varying degrees of success) by Parsons, Brittan, and Beck, among

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Gerhardt. (Georg Olms, 1960-61: reprint of 1875-1890 Berlin edition), 2:282). These considerations seem to me to provide some answer to Friedman's objections to 'anti-formalist' views like the one put forward here, that is, the objection that they cannot explain 'the role of Kant's conception of the syntheticity of mathematics in motivating his rejection of the dogmatic metaphysics of the Leibnizean-Wolffian philosophy' (Friedman, 104). This issue is discussed in detail in my doctoral dissertation, 'Mathematics, Metaphysics and Intuition in Kant' (Harvard, 1996).

others.[11] To support his own interpretation, Friedman offers an argument against these alternative interpretations which emphasize the syntheticity of geometrical truths *apart from* the syntheticity of geometrical *inferences*. I shall begin, then, by considering this argument in order to make clearer exactly what one is committed to on such a view. The basic claim with which Friedman takes issue is the claim that the role Kant assigns to pure intuition arises out of some kind of anti-formalism: pure intuition is supposed to provide mathematical concepts with content, thereby distinguishing the objectively true geometry from other logically possible (but empty) systems. The main support for this view comes from a passage in the Postulates of Empirical Thought (A220-1/B268), where Kant says:

> It is, indeed, a necessary logical condition that a concept of the possible must not contain any contradiction; but this is not by any means sufficient to determine the objective reality of the concept, that is, the possibility of such an object as is thought through the concept. Thus there is no contradiction in the concept of a figure which is enclosed within two straight lines, since the concepts of two straight lines and of their coming together contain no negation of a figure. The impossibility arises not from the concept in itself, but in connection with its construction in space, that is, from the conditions of space and of its determination.

The claim is then that construction in pure intuition exhibits the objective reality of geometrical concepts by showing the possibility of an object corresponding to the concept. For example, it establishes that although the concept of a figure enclosed between two straight lines is logically possible, it is not possible in some stricter sense. This reading depends on there being, first, logically possible mathematical concepts which are impossible in a narrower sense, and second, a sense of possibility corresponding to constructibility in pure intuition.

Friedman objects to both of these claims, arguing that there is in Kant neither a sense of possibility in which non-Euclidian geometries are possible, nor a sense of possibility corresponding to constructibility in pure intuition. I'll consider these objections in turn.

Friedman's argument against the claim that Kant recognizes logically possible mathematical concepts which are not possible in some less

11 See for example, Lewis White Beck, 'Can Kant's Synthetic Judgments be Made Analytic?' in Robert Paul Wolff, ed., *Kant: A Collection of Critical Essays* (London: MacMillan 1968) 3-22; Gordon Brittan, *Kant's Theory of Science* (Princeton: Princeton University Press 1978); Charles Parsons, 'Kant's Philosophy of Arithmetic,' in S. Morgenbesser, P. Suppes, and M. White, eds., *Philosophy, Science and Method: Essays in Honor of Ernest Nagel* (New York: St. Martin's Press 1969), reprinted in *Mathematics in Philosophy* (Ithaca: Cornell University Press 1983) 110-51.

inclusive sense rests largely on Kant's claim that 'I cannot think a line except by drawing it in thought' (B154). Just as Friedman takes the argument for the intuitive nature of our representation of space to rest ultimately on our knowledge of geometry, he suggests that this claim about the possibility of representing spatial concepts also rests on geometrical considerations. The reason that Kant says that we can only think a line by drawing it is that 'only this representation permits me to use the concept of line in mathematical reasoning ... where properties like denseness and continuity play an essential role' (Friedman, 80-1). In other words, any representation of a line *for mathematical purposes* must have at least these properties which, as we've seen, are not capturable in monadic logic. Such a representation must therefore be intuitive, drawn or drawable in Euclidian space. So a non-Euclidian line cannot be represented and thus, Friedman concludes, 'the very idea of a non-Euclidian geometry is quite impossible' (Friedman, 82).

While it may be true, as Friedman says, that only the intuitive representation of a line is adequate for mathematical reasoning, it by no means follows that there can be 'no idea' of a non-Euclidian line or figure. Since Kant admits that we may possess empty concepts (concepts for which there can be no corresponding intuition), he clearly cannot hold that the criteria for possessing a concept satisfy the standards of mathematical rigor. At most, what is required is that we be able to entertain the possibility of other spaces; there need be no determinate conception of what that space would be like. Kant explicitly recognizes the possibility of other creatures with different modes of intuition (e.g., A27/B43, B148-150, Inaugural Dissertation §1). But while we can have no determinate idea of their experience, we can imagine that it is unlike ours. We can, he says, *represent* an object of a non-sensible intuition negatively 'through all the predicates which are implied in the presupposition that it has none of the characteristics proper to sensible intuition' (B149); thus we can *represent* it as not extended or in space, as not enduring through time, as not capable of change, etc.

> But there is no proper knowledge if I thus merely indicate what the intuition of an object is *not*, without being able to say what it is that is contained in the intuition. For I have not then shown that the object which I am thinking through my pure concept is even so much as possible, not being in a position to give any intuition corresponding to the concept, and being able only to say that our intuition is not applicable to it.

What is important here is that Kant allows that we can indeed *represent* such an object by listing certain (negative) 'predicates.' In a similar way, I would suggest, we can *represent* a figure enclosed between two straight lines; we needn't be able to imagine what it would be like.

Moreover, Kant has the resources within his account of the mathematical method to explain the formation of concepts of non-Euclidian figures. The distinction he draws in his lectures on logic between real and nominal definitions allows, I think, for the possibility that one can entertain a mathematical concept in abstraction from the conditions of pure intuition. A nominal definition, according to Kant, is 'that distinct concept which suffices for differentiation of a thing from others'; a real definition is 'that distinct concept which suffices for cognizing and deriving everything that belongs to the thing.'[12] In the Discipline of Pure Reason in its Dogmatic Employment, Kant refers to the 'mere' definition which contains 'what I am actually thinking' in the concept, and which issues in analytic propositions (A718/B746). Similarly, he goes on to say that it would be futile for the mathematician to philosophize upon the triangle, to think about it discursively, for 'I should not be able to advance a single step beyond the mere definition, which is what I had to begin with.' In order to gain mathematical knowledge, he continues, 'I must pass beyond this definition to properties which are not contained in this concept *but yet belong to it.*' The only way to do this is to 'determine my object in accordance with the conditions either of empirical or of pure intuition,' i.e., to construct it.

This reference to the *mere* definition as opposed to construction might seem to conflict with the importance of the role which Kant assigns to definitions in the mathematical method, and in particular, with his claim just a few pages later that mathematical definitions *are* constructions of concepts (A730/B758). This apparent conflict can, I think, be eliminated by appeal to the distinction between real and nominal definitions. The real mathematical definition is the determination of the object in accordance with the conditions of pure intuition, the construction of the concept. This is not to say that one cannot give a mere nominal definition in accordance with concepts alone: such a definition would be something like a collection of characteristic marks.[13] In this way, one can form the concept of a figure enclosed by two straight lines simply by 'combining' the relevant concepts, in the same way that one can form the concept of, say, a slumbering monad: both of these have 'logical essences' (Ak.9:143). However, as soon as one attempts to employ the concept in mathematical thought, to gain *knowledge* of the concept, it becomes clear

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12 Ak.24:919-20. See also Ak.24:268ff and Ak.9:143-4. Kant's theory of definition deserves much more attention than can be given to it here.

13 It should be noted that the nominal definition is, strictly speaking, not a definition at all for Kant.

that such an object cannot be determined in accordance with the conditions of pure intuition — it cannot be constructed. Thus when Kant says 'I cannot think a line without drawing it in thought, or a circle without describing it,' he is referring to a narrower use of the understanding than the purely logical. In other words, one can entertain the concept of a line all of whose points are equidistant from one and the same point without actually describing a circle. But one cannot employ the concept in mathematical thought without describing it, for the reasons that Friedman makes clear.

Kant is more explicit about this in the B edition of the Transcendental Deduction, where he says that 'to *know* anything in space (for instance, a line), I must *draw* it' (B137-8). In this context, Kant is more concerned with the application of the synthetic unity of apperception than with the intuitive nature of the representation — in fact, the point here is to show that space as the form of intuition only supplies the manifold of a priori intuition for a possible knowledge, and cannot yield knowledge by itself — but the distinction between *thinking* an object and *knowing* it is all we need concern ourselves with, and it is precisely this distinction which Kant goes on to draw at B146.

On this way of understanding it, then, the distinction between real and nominal definitions corresponds to the two elements that Kant says we demand in every concept:

> first, the logical form of a concept (of thought) in general, and secondly, the possibility of giving it an object to which it may be applied. In the absence of such object, it has no meaning and is completely lacking in content, though it may still contain the logical function which is required for making a concept out of any data that may be presented. (A239/B298)

Apart from this relation to the data for a possible experience, Kant goes on, the concept has no objective validity but is a 'mere play of imagination or understanding.' It seems significant that he then takes as examples the concepts of mathematics: 'they would mean nothing if we were not always able to present their meaning in appearances'; further, 'the mathematician meets this demand by the construction of a figure.' If these elements, the logical form and the possibility of giving it an object, can be distinguished in mathematical concepts, then presumably we can form a concept of a figure which has the logical form of a concept, but which cannot be given an object. This is all that the proponent of the anti-formalist view is committed to.

Friedman's objection, however, assumes that the anti-formalist attribute to Kant a picture of competing rigorous axiomatic systems of geometry, with pure intuition providing objects for one but not others. Indeed, he characterizes the view as one according to which Kant takes pure intuition to provide a model for Euclidian geometry. Of course, if one demands that recognizing the possibility of non-Euclidian geometry requires possessing a fully developed mathematical theory based on inferences, then indeed, Kant's theory of geometrical proof forbids this possibility because of the role of Euclidian construction in inference. But the connection between pure intuition and objective reality does not require such a reading. The claim is merely that it is in virtue of the form of intuition that Euclidian geometry is true, and thus that concepts of Euclidian figures can be constructed and concepts of non-Euclidian figures, like the figure enclosed by two straight lines, cannot.

Friedman's second (related) objection is that there is no sense of possibility corresponding to constructibility in pure intuition, and so it can't be the task of pure intuition to confer objective reality on certain logically possible concepts. He bases this objection on Kant's claim (at A239/B298-9) that a pure intuition 'can acquire its object, and therefore objective validity, only through the empirical intuition of which it is the mere form.' As examples, Kant considers the geometric principles that space has three dimensions and that between two points there can be only one straight line, which, he says, 'would mean nothing were we not always able to present their meaning in appearances, that is, in empirical objects.' Friedman thus concludes that it is not construction in pure intuition but rather *empirical* intuition that provides mathematical concepts with objective reality, since pure intuition itself must look to empirical intuition for its object and objective validity. So while construction in pure intuition gives us knowledge of objects 'in regard to their form,' it does not yet show the *possibility* of those objects: 'whether there can be things which must be intuited in this form is' as Kant says, 'still left undecided' (B147). To demonstrate that possibility requires showing

> that space is a formal *a priori* condition of outer experiences, that the formative synthesis through which we construct a triangle in imagination is precisely the same as that which we exercise in the apprehension of an appearance, in making for ourselves an empirical concept of it.... (A224/B271)

This is the notion of possibility which Kant elaborates in the Postulates of Empirical Thought: what is possible is 'that which agrees with the formal conditions of experience, that is with the conditions of intuition *and* of concepts' (A218/B265). Thus the demonstration of this kind of possibility is, as Friedman points out, the task of transcendental philosophy: it alone establishes the requisite supposition (stated at B147) that 'there are things which allow of being presented to us only in accordance with the form of that pure sensible intuition.'

But does this pose the problems for the view under consideration that Friedman thinks it does? For one thing, it has to be reconciled with clear statements on Kant's part of a connection between construction of concepts and their objective reality. For example, in a work from 1790, 'On a discovery according to which any new critique of pure reason has been rendered superfluous by an earlier one,' Kant addresses Eberhard's claim that mathematicians have 'completed the delineation of entire sciences without saying a single word about the reality of their object' (Ak.8:190). To illustrate his point, Eberhard describes Apollonius as having constructed the entire theory of conic sections without showing how it can be applied, 'despite the fact that the reality of the entire theory rests on this' (*The Kant-Eberhard Controversy*, 19). In his reply to this alleged counterexample to his view, Kant describes Apollonius' procedure as follows:

> Apollonius first constructs the concept of a cone, i.e., he exhibits it a priori in intuition (this is the first operation by means of which the geometer presents in advance the objective reality of his concept). He cuts it according to a certain rule ... and establishes a priori in intuition the attributes of the curved line produced by this cut on the surface of the cone. Thus he extracts a concept of the relation in which its ordinates stand to the parameter, which concept, in this case, the parabola, is thereby given a priori in intuition. Consequently, the objective reality of this concept, i.e., the possibility of the existence of a thing with these properties, can be proven in no other way *than by providing the corresponding intuition*. (Ak.8:191)

Here Kant seems to deny explicitly that anything over and above the construction of the concept in pure intuition (such as application of the concept to an empirical object) is required to establish its objective reality. How are these two positions to be reconciled? I now want to sketch one possible way.

It is clear, as Friedman says, that for Kant, construction in pure intuition alone does not establish the real possibility of objects corresponding to mathematical concepts. I want to suggest instead that this is achieved indirectly, by means of the argument of the Axioms of Intuition. There, Kant establishes the principle of these axioms, which he calls 'the transcendental principle of the mathematics of appearances' (A165/B206), that is, that all intuitions are extensive magnitudes. Having shown already that all intuitions are in space and time, he argues now that 'they must be represented through the same synthesis whereby space and time in general are determined' (A162/B203). It follows from this that 'empirical intuition is possible only by means of the pure intuition of space and of time.' In this way, then, transcendental philosophy establishes that 'what geometry asserts of pure intuition is therefore undeniably valid of empirical intuition' (A165/B206): for any figure constructed in pure intuition, there is therefore a possible empirical object with those spatial properties. So pure intuition *does* give us knowledge of objects 'in regard to their form,' as Kant says (B147): it exhibits the properties that objects must have if they are to be in conformity with the form of intuition. It is in this sense that construction is concerned with possibility. It seems clear that the construction in pure intuition of a triangle whose angles sum to 180 degrees reveals that such a figure is possible in a sense in which a figure enclosed by two straight lines (i.e., a figure which is not so constructible) is not. This is not to say that the constructed triangle itself has objective reality: construction does not establish the real possibility of some kind of mathematical object. We might say, rather, that the proof in the Axioms of Intuition establishes that construction in pure intuition reveals the form of (really) possible objects of empirical intuition, thus of objects of possible experience.

Thus mathematical concepts earn their objective reality derivatively: in establishing that whatever geometry asserts of pure intuition is valid of empirical intuition, the objective reality of geometrical concepts which are constructible in pure intuition is thereby also established. As Kant says at A733/B761: 'the possibility of mathematics must itself be demonstrated in transcendental philosophy.' In other words, once the transcendental facts are given, and the objective validity of pure intuition is established, we can see that construction in pure intuition in turn confers objective reality on mathematical concepts. It is irrelevant whether or not the mathematician alone provides the complete demonstration. Indeed, there is thus a clear distinction between constructibility and existence: mathematicians are not concerned with real existence at all, the way philosophers are. As Thompson has put it,[14] existence questions *within* mathematics are really questions of constructibility, and they are answered by pure intuition. Questions of real existence are answered only by empirical intuition. Mathematicians are still concerned with the objective reality of their concepts though: only this ensures that mathematics is not 'a mere play of imagination' (A239/B298).

There seems to be some textual support for something like this way of looking at the question of possibility. Wherever Kant explains that application to empirical intuition is required to establish the objective validity of pure intuition, he attaches this as a *condition* on the role of pure intuition, a condition which transcendental philosophy shows to be met. For example, at A239-40/B299, Kant says that the principles and constructions of geometry 'would mean nothing were we not always able to present their meaning ... in empirical objects.' The argument of the Axioms of Intuition is sufficient to show that we are always able to

14 Manley Thompson, 'Singular Terms and Intuitions in Kant's Epistemology,' *Review of Metaphysics* **26** (1972-3) 314-43

present their meaning in empirical objects. Similarly, at A156/B195, Kant emphasizes that 'even space and time ... would yet be without objective validity, senseless and meaningless, if their necessary application to the objects of experience were not established.' More explicitly, he continues:

> Although we know *a priori* in synthetic judgments a great deal regarding space in general and the figures which productive imagination describes in it, and can obtain such judgments without actually requiring any experience, yet even this knowledge *would be* nothing but playing with a mere figment of the brain, *were it not* that space has to be regarded as a condition of the appearances which constitute the material for outer experience. (A157/B196, my emphasis)

Kant seems here to be saying that we can obtain knowledge of geometrical propositions — hence objectively valid true judgments — without requiring any experience. The condition for the objective reality of the concepts involved in these judgments therefore cannot be that their objects be presented in empirical intuition; it is rather that space is the condition of the appearances which constitute the material for outer experience. That is, the condition for the objective reality of constructible mathematical concepts is the objective validity of the concept of space. That condition, again, is shown to be satisfied by transcendental philosophy. Thus Kant concludes that the pure synthetic judgments of mathematics relate 'only mediately to possible experience, or rather, to the possibility of experience; and upon that alone is founded the objective validity of their synthesis' (A157/B191). The relation is only mediate because the 'objective validity of their synthesis' is established by the constructibility of the concepts in pure intuition, and not by their being given in empirical intuition. For this reason, 'mathematical concepts are not therefore, by themselves knowledge, except on the supposition that there are things which allow of being presented to us only in accordance with the form of that pure sensible intuition.' We can therefore conclude that construction in pure intuition, *given* the results of transcendental philosophy, establishes the real possibility of (empirical) objects corresponding to the concepts. This is quite different from saying, as Friedman does, that the real possibility of, say, a triangle 'depends entirely on *applied* mathematics' (Friedman, 102); more importantly, it does not license his conclusion that 'considerations of objective reality and real possibility can therefore not themselves explain Kant's doctrine of pure intuition' (ibid.).

It is this role of construction in pure intuition as conferring objective reality on mathematical concepts that reflects Kant's anti-formalism. The difference between this and Friedman's reading is that according to the latter, geometry is constrained by pure intuition because only intuition makes the representation of mathematical concepts possible. If a purely conceptual representation were possible, it seems, there would be no such constraint. I am suggesting that mathematics is constrained by pure intuition because only that can provide its concepts with objective content, thereby ensuring that it is not a mere play of imagination. This condition would remain even if a purely conceptual representation of mathematical concepts of the kind which Friedman seems to envisage were possible.

This is not to deny that intuition does play the inferential role that Friedman describes. As Friedman has shown, it must be called on to guarantee that we can indeed perform the steps of the constructions involved in Euclidian proofs (for example, extending the line segment by the second Euclidian postulate), that is, to perform the role that existence assumptions play in the kind of axiomatization unavailable to Kant. But to understand what this means we have to ask what it can mean to *guarantee* that we *can* perform these steps. It is not enough merely to guarantee that the step is in accordance with the postulates, but the postulates themselves have to be 'guaranteed.' I take this to be the point of the following passage from the Observation on the Second Antinomy (A439/B467):

> For however evident mathematical proofs may be, [the monadists] decline to recognize that the proofs are based upon insight into the constitution of space, insofar as space is in actual fact the formal condition of the possibility of all matter. They regard them merely as inferences from abstract but arbitrary concepts, and so as not being applicable to real things.[15]

What Kant is claiming is that it is immediately certain and evident (A733/B761), in virtue of our 'insight into the constitution of space,' that what Euclid's postulates assert can be done can indeed be done, thus that they are true of space. For example, as I suggested earlier, Kant seems to think that the unboundedness of space guarantees that we can extend a line indefinitely. In addition, this brings out the importance to this reading of Kant of the *immediacy* of intuition, in the sense of what Parsons describes as 'phenomenological presence to the mind, as in perception' (Parsons, 'The Transcendental Aesthetic,' 66). It is precisely this phenomenological aspect of intuition which, I argued, Friedman does not account for in his reading of the Metaphysical Exposition. It also plays an important role in Kant's theory of geometry in providing a notion of

15 This also seems to support the view that Kant's objection to the Leibniz-Wolff account of mathematics is philosophical (anti-formalist) and not mathematical (based on the impossibility of proving theorems with only monadic logic); cf. n. 10 above.

intuitive *evidence* that explains the immediate certainty of geometrical knowledge, which, again, Friedman's account does not capture.

So it seems that while modern formal representations of denseness, continuity, or indefinite extendibility may perform the inferential role of constructions, there would still be, for Kant, the question of grounding these formal representations, of guaranteeing their objective reality. Otherwise, all that is guaranteed by the appeal to an existence axiom is that the inference is in accordance with the axioms. To return to our original example, in a rigorous proof, the existence of the point of intersection of two circles is 'guaranteed' by the continuity axiom. For Kant, there would still be a role for pure intuition in grounding or justifying that axiom. This is the sense in which, for Kant, the geometer expressly grounds the possibility of his task on the original representation of space.

Conclusion

In the first part of this paper, I tried to argue against Friedman that it is the task of the Metaphysical Exposition to provide grounds, independent of geometrical concerns, for taking the space in which objects are given to be as Euclidian geometry describes it: in particular, for the infinity of space. There, the arguments for the nature of our representation of space depended on certain data of experience, which require that space be an infinite given magnitude. Geometrical knowledge is then explained as knowledge of that form of intuition which is derived from the conditions of the possibility of experience. Friedman recognizes that there remains, on his view, 'a serious question about Euclid's axioms' (Friedman, 95). As I've tried to suggest in the second part of this paper, this is at the heart of the matter. For Kant, there is some content to the axioms, something in virtue of which they are true, and we have immediate epistemological access to it. On Friedman's view, where geometry is taken to be a 'form of rational argument and inference,' the status of the underlying assumptions is left an open question. Indeed, it seems to me to be a particularly important question if geometry is to be taken as the data for Kant's transcendental method, as Friedman argues it is. More importantly though, I have suggested that this view of geometry as a *form* of argument and inference is in fact the sort of formalist view of mathematics which Kant is arguing *against*. His main task is to show that mathematical proofs are not merely 'inferences from abstract but arbitrary concepts'; he does this by arguing that they are 'based on insight into the constitution of space.' This task has a modern analogue in attempts to justify the axioms of set theory by appeal to some fundamental conception of set or class such as, for example, the iterative conception. It is in this way that, as I suggested at the beginning, Kant is concerned with issues which are of importance independently of the limits of the logic and geometry of his time; indeed, it could be argued that these issues have become more pressing with the development of modern logic, but this is not the place to argue that.[16]

Received: November, 1995
Revised: October, 1996
Revised: April, 1997

16 I am grateful to John Carriero, Janet Folina, Warren Goldfarb, Michael Hallett, Alison Laywine, Charles Parsons, and two anonymous referees for this journal for comments on various drafts of this paper, to Peter Clark for making me write it, and to the Social Sciences and Humanities Research Council of Canada for generous support of my doctoral studies, during which this paper was written.